Epistemic game theory and logic

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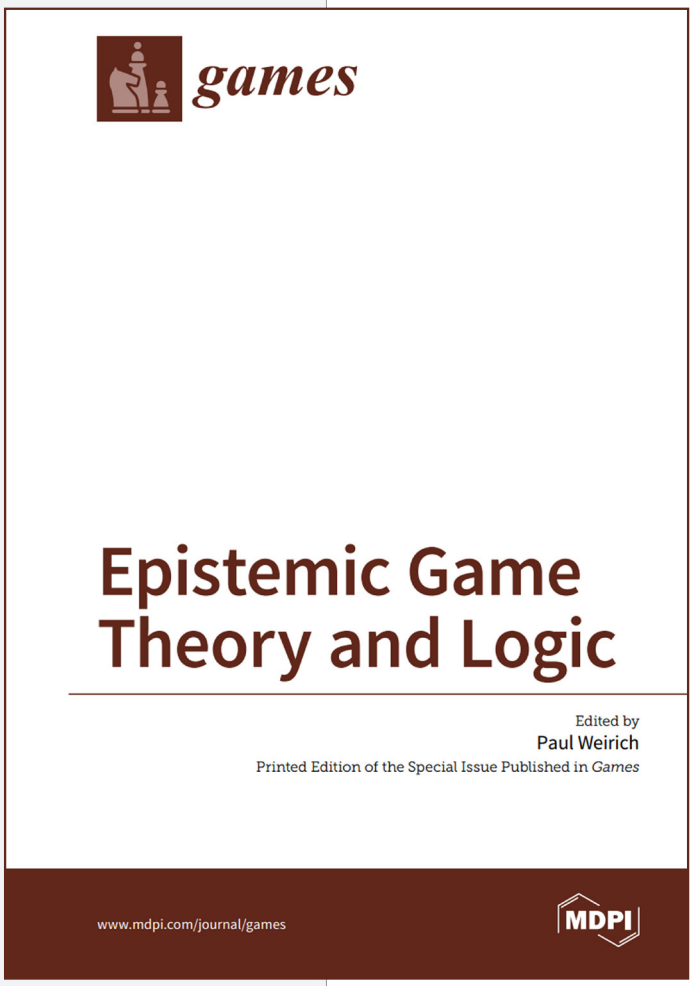
Paul Weirich, University of Missouri USA, edited the Printed Edition of the Special Issue Epistemic Game Theory and Logic, a collection of papers published online on MDPI between 2016 and 2017. The issue is divided into four sections: Economics, Computer Science, Philosophy and Psychology.

We are pleased to publish, for divulgation purposes, the preface to the Special Issue and sincerely thank the author for allowing us to reprint it (Note of the Italian Editor).

Preface

Epistemic game theory and the systems of logic that support it are crucial for understanding rational behavior in interactive situations in which the outcome for an agent depends, not just on her own behavior, but also on the behavior of those with whom she is interacting. Scholars in many fields study such interactive situations, that is, games of strategy.



Epistemic game theory presents the epistemic foundations of a game's solution, taken as a combination of strategies, one for each player in the game, such that each strategy is rational given the combination. It considers the beliefs of the players in a game and shows how, along with the players' goals, their beliefs guide their choices and settle the outcome of their game. Adopting the Bayesian account of probability, as rational degree of belief, it yields Bayesian game theory. Epistemic game theory, because it attends to how players reason strategically in games, contrasts with evolutionary game theory, which applies to non-reasoning organisms such as bacteria.

Logic advances rules of inference for strategic reasoning. It contributes not just standard rules of deductive logic, such as modus ponens, but also rules of epistemic logic, such as the rule going from knowledge of a set of propositions to knowledge of their deductive consequences, and rules of probabilistic reasoning such as Bayesian conditionalization, which uses probabilities conditional on receiving some new evidence to form new non-conditional probabilities after receiving exactly that new evidence.
Perea (2012) offers an overview, and Weirich (1998) shows how principles of choice support solutions to games of strategy.
The papers in the special issue came in response to the journal's call for papers. Diversity of perspectives was a goal. The papers include four by economists, one by computer scientists, three by philosophers, and one by a psychologist. They display a variety of approaches to epistemic game theory and logic. The following paragraphs briefly describe the topics of the papers, grouped according to discipline and within a discipline according to date of publication.

Economics

Perea and Kets (2016), "When Do Types Induce the Same Belief Hierarchy?" Higher-order beliefs, that is, beliefs about beliefs, play an important role in game theory. Whether one player decides to make a certain move may depend on the player's beliefs about another player's beliefs. A representation of a player's higher-order beliefs may use a type structure, that is, a set of player types that characterize a player's uncertainty. This paper investigates conditions under which two type structures may represent the same hierarchical beliefs. Bonanno (2016), "Exploring the Gap between Perfect Bayesian Equilibrium and Sequential Equilibrium." This paper uses methods of belief revision to characterize types of equilibrium in a game with sequences of moves. It assumes that belief revision is conservative or minimal and then obtains two types of equilibrium that in a certain sense are intermediate between perfect Bayesian equilibrium and sequential equilibrium. It argues that refinements of subgame-perfect equilibrium in extensive-form games should attend to principles of belief revision, in particular, iterated belief revision. Asheim et al. (2016), "Epistemically Robust Strategy Subsets." This paper examines the epistemic foundations of set-valued generalizations of strict Nash equilibrium. It explores, in particular, epistemic robustness, or support by epistemic considerations, of sets of strategy profiles taken as solutions to games in which players have doubts about the rationality of players, and so doubts about the strategy a player adopts in response to other players. It classifies a player according to her doxastic state, characterized by a probability distribution over the other players' strategies and types. Yang and Harstad (2017), "The Welfare Cost of Signaling." This paper treats learning about differences among agents from observations of the agents' behavior. It investigates, in particular, separating workers according to skill using signals an employer elicits from the workers. A credible signal may involve a transfer of resources instead of consumption of resources, and so not impose a welfare cost on society. The paper shows specifically that an employer's charging a job application fee may generate such credible signals, and it explores the robustness of its results given variation in factors such as a job applicant's aversion to risk.

Computer science

Chen and Micali (2016), "Leveraging Possibilistic Beliefs in Unrestricted Combinatorial Auctions." This paper addresses a topic in economics, namely, the design of auctions, with special attention to methods of computing a rational strategy given the design's adoption. The design of the auction

aims to guarantee an amount of revenue, dependent on bidders' "possibilistic beliefs" about bidders' valuations of an item up for auction. A bidder's possibilistic beliefs settle the valuations of other bidders that are epistemically possible for the bidder, that is, may be correct for all the bidder knows. The paper describes the benefits of its auction design for a bidder's privacy concerning her valuations, for computationally simplifying a design's implementation, and for limiting collusion among bidders.

Philosophy

Hédoin (2016), "Community-Based Reasoning in Games: Salience, Rule-Following, and Counterfactuals." This paper argues for a representation of a game that includes, when the players know that they come from the same community, the players' knowledge of this fact. Their knowledge of their cultural ties assists their predictions of their behavior; it explains a focal point of their game, that is, a strategy profile that the players expect the players to realize. The paper's representation of a game includes an epistemic model grounding applications of epistemic logic that yield conclusions about the players' behavior. The model covers not only actual behavior, but also behavior in hypothetical situations that do not arise during a play of the game. The paper examines the literature on following rules because a culture's rules govern counterfactual as well as actual situations. Cozic (2016), "Probabilistic Unawareness." This paper considers whether an agent in a game, for a proposition relevant to the game's solution, believes the proposition and whether, if the proposition is true, the agent is aware or unaware of its truth. An agent may be unaware of a proposition's truth because he fails to entertain it. The paper uses partial (and so probabilistic) belief as opposed to full belief in its representation of an agent's beliefs. It extends doxastic logic to obtain a version of probabilistic logic, namely, a formal system with an explicit representation of partial belief together with an explicit representation of awareness. Its main result is a soundness and completeness theorem, using its semantics of partial belief and awareness, for its probabilistic extension of doxastic logic. Sperry-Taylor (2017), "Strategy Constrained by Cognitive Limits, and the Rationality of Belief Revision Policies." This paper investigates the effect of cognitive limits on the strategic reasoning of the players in a game. It examines how as a game progresses, imperfect agents playing the game revise their beliefs (taken as probability assignments or assignments of credence to propositions), and how their methods of belief revision affect their choices and the outcome of their game. Cognitive limits may excuse an agent's failure to consider all the options in a decision problem and may excuse an agent's failure to consider all her strategies for playing a game. The paper advances principles of rationality that govern belief revision by cognitively limited agents who in games entertain some contingencies only as they arise.

Psychology

Suleiman (2017), "Economic Harmony: An Epistemic Theory of Economic Interactions." This paper maintains that some under-studied epistemic factors, in particular, an agent's aspirations, influence the agent's reasoning in a game of strategy. It defines an agent's subjective utility assignment to the possible outcomes of an option in a decision problem as a quantitative representation of the agent's mental attitudes to the outcomes, roughly, levels of satisfaction from the outcomes, so that the utilities of the outcomes form a ratio scale. Then it proposes that the value of an outcome for an agent is the ratio of the utility for the agent of the payoff the outcome yields divided by the utility for the agent of the payoff to which the agent aspires. As a solution to a game, it proposes a combination of strategies, with

exactly one strategy for each player that yields an outcome with the same utility for all the players. The paper supports its proposals by noting their agreement with experimental data concerning behavior in games such as the Ultimatum Game and the Common Pool Resource Dilemma. This Special Issue gathers recent scholarship in several disciplines treating epistemic game theory and logic. Some articles propose new ways of representing the mental states of the players in a game or new ways of making inferences about their mental states, some strengthen the logical foundations of epistemic game theory or extend its scope, and some present applications of epistemic game theory. The articles show the vitality and significance of this area of research.

* Paul Weirich is a Curators' Distinguished Professor in the Philosophy Department at the University of Missouri. He is the author of Equilibrium and Rationality (Cambridge, 1998), Decision Space (Cambridge, 2001), Realistic Decision Theory (Oxford, 2004), Collective Rationality (Oxford, 2010), and Models of Decision-Making (Cambridge, 2015).